Universality of dynamic scaling for avalanches in disordered Ising systems

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Dynamic scaling for driven disordered systems is investigated in some disordered Ising models. Using Monte Carlo simulation, we find that avalanches in both random-field and random-bond Ising models follow dynamic power-law scaling in short times, and the scaling relations are universal for the systems studied. The probability distribution of the dynamic scaling exponent θ is found to have two peaks centered at θ_1 and θ_2. The short-time dynamic exponent θ_1 is invariant and universal for all avalanches while the exponent θ_2 depends on the strength of disorder. The analytical result for the early stage evolution of breakdown process in the random-field Ising model is obtained using mean-field approximation. Short-time dynamic scaling is also confirmed.

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I. INTRODUCTION

Effect of disorder on phase transition has attracted a lot of attention in the past two decades [1–3]. Because defects cause many local energy minima in the free energy landscape of the system, they play important roles in phase transitions and critical phenomena. For example, impurities can change the critical temperature and the critical behaviors, or completely destroy the order phase, i.e., disorder-induced phase transition [3]. When disorder is weak, the energy barriers among local energy minima may be low and disorder can be treated as a perturbation to the configuration space. When disorder is strong, the energy barriers are so high that the system is trapped in the metastable configurations. Thermal fluctuation cannot activate the system from the metastable state in finite time scales. In this case, theoretical and simulation treatments to the disorder-induced phase transition and critical phenomena have encountered tremendous difficulties [3]. To avoid these difficulties, driven disordered system at zero temperature, i.e., disordered system driven by an external field, has been employed and extensively studied [4,5]. Avalanches are usually observed during the field-driven phase transition in disordered system. In some well-defined models, such as random-field Ising models (RFIM), the critical exponents obtained by scaling for avalanches are found to be consistent with those determined by thermal equilibrium critical scaling.

One of the well-known examples of avalanches in driven disordered system is the Barkhausen jump during the magnetization reversal process. When disordered ferromagnets are subjected to a slowly varying magnetic field, the avalanches of local domain reversals form, which lead to the Barkhausen jumps in the magnetization curve. Experimental results have shown that the avalanches of domains with different sizes are observed to obey scale invariance in space, time, and frequency domain. The distribution of avalanche sizes, the duration times and the power spectral density show power-law behaviors [6–8]. In theoretical studies, some of the disordered Ising models that are employed to describe the Barkhausen noise, have also demonstrated that the distribution of avalanches in hysteresis loop exhibits statistical scaling behaviors when the strength of the disorder is tuned to the critical value. The scaling exponents are related to the critical exponents of disorder-driven phase transitions in the systems without applied field [10,11]. However, those scaling exponents strongly depend on disorder strengths, initial states, and driving rates; and there is no universal scaling relation for these static and dynamic exponents [12]. Furthermore, most of the experimental [6–8] and theoretical [9–11] works on Barkhausen avalanches in disordered ferromagnets have focused on their stationary or metastable behaviors, e.g., the statistical distribution of signal strength (avalanche size) and duration time (avalanche time) of Barkhausen noises. There is little knowledge about the dynamical properties of Barkhausen avalanches [12,13]. For example, how avalanches evolve remains unknown and how the driving rate of applied field affects the avalanche kinetics has not been clarified in most of the theoretical works [13]. The question concerning the nature of the dynamics of Barkhausen avalanche process, especially whether different avalanches have different dynamic behaviors or they could be described by the same universal dynamic scaling, has not been clear so far.

In a recent study [14] on a two-dimensional (2D) RFIM, we find that the largest avalanche grows with power law in time when the system parameters are tuned to their critical values, i.e., at the critical point (D_c, H_c) of the model, where D_c is the critical disorder strength of the random field and H_c is the critical applied field that triggers the largest avalanche [11]. When the strength of disorder D is varied below D_c, the largest avalanche is found to evolve in two distinct stages. In short times, it behaves like a diffusive process, while at late times the avalanche shows nucleation and growth behavior that can be described by the kinetic theory of first-order phase transition [14].

There are two important issues in the previous work that remain unresolved for the dynamic scaling of avalanches in driven disordered systems. The first one is whether the dynamical scaling is universal in higher dimensions and for systems with quenched-in disorder, such as the models with random interactions. Second, it is of great interest for us to look at the dynamic behavior of all avalanches other than only the largest one. Specifically, we need to address the following question: Whether the dynamics of avalanches of all sizes is homogeneous or heterogeneous, i.e., whether all avalanches evolve like the largest one.

These unanswered questions provide the major motivation for this work. Our previous work gave a limited account of the effects of disorder on the dynamic process in a 2D random-field Ising model [14]. The purpose of this paper is to give more general and more extensive answers to the aforementioned questions using different model systems and Monte Carlo simulations.

This paper is organized as follows. We shall briefly review model systems for driven disordered system and the dynamic scaling for avalanches in the following section. In Sec. III, we demonstrate through numerical simulations that the dynamic scaling is universal in higher dimensions and for the disordered model systems in which the hyperscaling relation is not valid [11]. In Sec. IV, we investigate the dynamic heterogeneity of avalanches in the random-bond Ising model (RBIM). In Sec. V, analytical results are obtained by using mean-field approximation to RFIM. The last section summarizes the results.

II. DYNAMIC SCALING FOR DRIVEN DISORDERED SYSTEMS

We consider a coarse-grained Ising model with quenched disorder,

$$\hat{H} = -\sum_{\langle ij\rangle} J_{ij}S_iS_j - \sum_i h_iS_i - H\sum_i S_i, \tag{1}$$

where $S_i = \pm 1$ are spin variables and $\langle ij\rangle$ denotes the summation extending over all nearest-neighbor spins. H is a homogeneous external magnetic field; h_i is an uncorrelated random field which has a Gaussian distribution; $\langle h_i\rangle = 0$ and $\langle h(x)h(x')\rangle = 2D\delta(x-x')$.

In this work, we consider two variations of the Ising model represented by Eq. (1): (a) random-field Ising model and (b) random-bond Ising model. For the random-field Ising model, we choose $J_{ij} = J$, and h_i is the random field as described above. For the random-bond Ising model, we choose $h_i = 0$. J_{ij} are the Gaussian random variables with the mean value J and standard derivation D. In both models, H and D are in units of J. The temperatures in both random-field and random-bond Ising systems described by Eq. (1) are kept at zero.

In the system described by Eq. (1), if the applied field H (e.g., magnetic field) is swept between $+\infty$ and $-\infty$, its conjugate order parameter M (e.g., magnetization) does not vary smoothly from $+1$ to -1. Instead, many avalanche processes can be observed during the switching of order parameter. There is a critical disorder strength D_c that characterizes this driven disordered system. When $D > D_c$, there are only small avalanches; while at $D \leqslant D_c$, there exists an infinite avalanche under the critical applied field H_c. The equilibrium distributions of avalanches are found to obey the power-law scaling relations at $D = D_c$ [9,11],

$$P(s) \sim s^{-a},$$

$$P(\tau_0) \sim \tau_0^{-b}, \tag{2}$$

$$A(f) \sim f^{-c},$$

where s and τ_0 are avalanche size and duration time respectively. $A(f)$ is the power spectral density of avalanche signal. a, b, and c are the scaling exponents. On the other hand, the critical scaling for the driven disordered system has the following relations [10]:

$$\Delta M \sim (D_c - D)^{\beta\nu}, \qquad \langle t_0\rangle \sim (D_c - D)^{-\nu z}, \tag{3}$$

where ΔM is the avalanche size and $\langle t_0\rangle$ is the duration time of the largest avalanche during the switching process.

We need to point out that Eqs. (2) and (3) are the results from stationary or metastable scaling. The scaling is analyzed when the avalanches remain unchanged or metastable. However, nonequilibrium dynamic scaling for avalanches could be more suitable to describe the driven disordered system and can be derived from the finite-size scaling hypothesis [15]. Based on this finite-size scaling relation, the time-dependent avalanche jump $m(t) \equiv [M(t) - M_c]/2$ is shown to satisfy the following relation:

$$m(L,t) \sim L^{-\beta/\nu} F(L/\xi(t)), \tag{4}$$

where $M(t) \equiv \langle (\Sigma S_i)/L^d\rangle$ is the total magnetization, and $\langle \cdots \rangle$ denotes average over the random-field configurations. M_c is the magnetization at the critical point (D_c, H_c). L is the lattice size and $\xi(t)$ is the nonequilibrium spatial correlation length of the flipped spins at time t. F is the scaling function. From Eq. (4), we found [14] that at the critical point (D_c, H_c), the dynamic evolution of the largest avalanche in short times obeys the following scaling relations:

$$\langle s(t)\rangle \sim t^{[d-\beta/\nu]/z} \sim t^{\theta} \tag{5}$$

and

$$\left.\frac{\partial \ln\langle s(t)\rangle}{\partial t}\right|_{r=0} \sim t^{1/\nu z}, \tag{6}$$

where $r \equiv (D_c - D)/D_c$. $s(t) \equiv L^d m(t)$ is the avalanche size. β and ν are the equilibrium critical exponents of the system. z is the dynamical critical exponent and θ is a new dynamic exponent that also characterizes the dynamics of the system.

III. UNIVERSALITY OF DYNAMIC SCALING FOR AVALANCHE EVOLUTION

Equations (5) and (6) are confirmed by the recent simulations of a 2D random-field Ising model [14]. The critical exponents β, ν, z, and the critical value D_c obtained from the dynamic scaling are found to be consistent with those determined from the stationary scaling relations [Eqs. (2) and (3)]. The new dynamic exponent θ is equal to $[d-\beta/\nu]/z$, or $1/\sigma\nu z$ [11], where exponent σ is defined from the spanning cluster. As it is known, the hyperscaling relation $d-\beta/\nu$

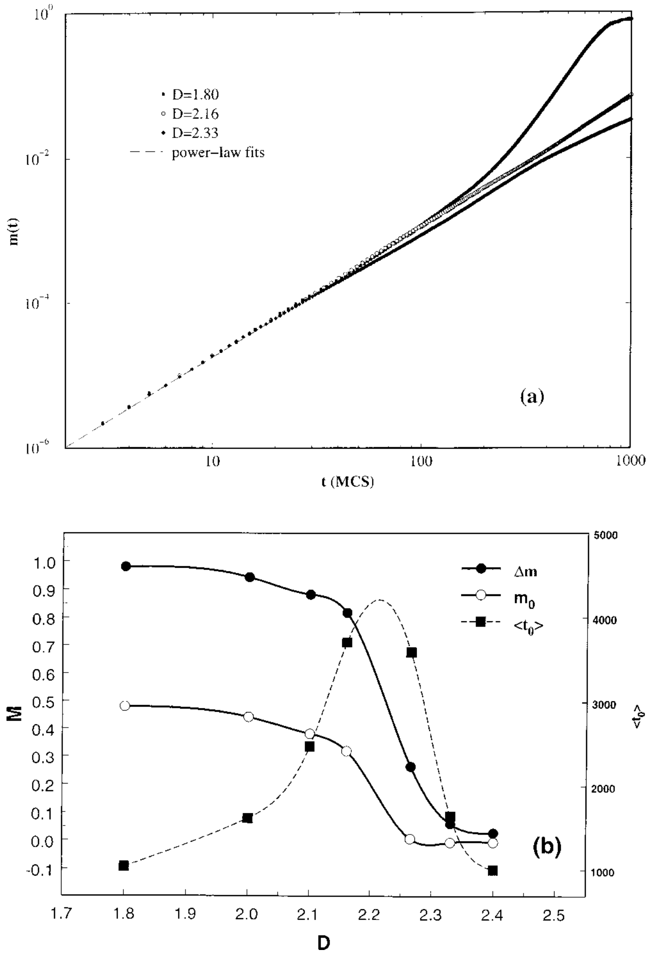

FIG. 1. Dynamic and stationary critical scaling for the largest avalanche in a three-dimensional random-field Ising model. System size is $L=128$. (a) Evolution of avalanches at different disorder strengths. The dashed line is the best power-law fit. The plots are in log-log scale. (b) The duration time $\langle t_0 \rangle$ of the largest avalanche, ΔM of the largest avalanche, and the initial magnetization m_0 at the largest avalanche.

$=1/\sigma\nu$, as well as possible disorder-driven phase transitions, are found to exist in 2D RFIM [11]. One of the questions that naturally arise from these results is *whether the dynamic scaling observed in* 2D RFIM *is universal in different models of disordered systems where the hyperscaling relation does not hold*.

We believe that the dynamic scaling and scaling relation $\theta=[d-\beta/\nu]/z$ are universal for Ising systems with uncorrelated impurities. The reasons are based on the following arguments. First, Eqs. (5) and (6) govern the dynamics of avalanche in short-time regime. At short times, $\xi(t)$ is still small compared with L and the spanning clusters or fractal-shaped avalanches have not formed yet. Then the avalanche size $s(L,t)$ is scaled with system size L^d, and thus has nothing to do with the spanning clusters. Second, for nonequilibrium dynamic process, the finite-size scaling relation [Eq. (4)] may not necessarily lead to the hyperscaling relation.

To demonstrate the above conjecture that the dynamic scaling for avalanches in Ising-type driven disordered system is universal, we choose to study two different model systems: a 3D random-field Ising model and a 2D random-bond Ising model. These systems provide the typical examples representing two scenarios for our questions. The 3D RFIM is known for not having the hyperscaling relation and 2D RBIM has different disordered nature, e.g., spin glass, compared with the RFIM.

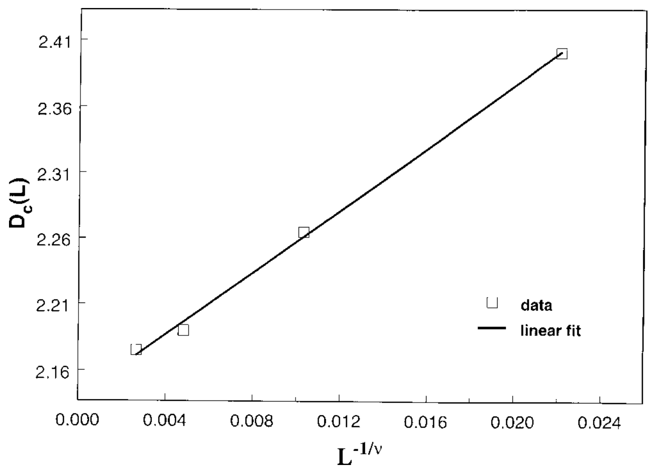

FIG. 2. Determination of $D_c(\infty)$ and critical exponent ν in 3D RFIM. $D_c(L)$ is the critical strength of disorder in a finite-size system. The solid line is a linear fit.

Additional advantages can also be gained from these two model systems. Since the exponents and critical phenomena of these systems are investigated previously [10,11] using stationary scaling relations [Eqs. (2) and (3)], we can compare these known results with those obtained from our dynamic scaling. This direct comparison could provide an additional justification for the universality of the dynamic scaling in the two model systems.

A. Algorithms for simulation study

We use Monte Carlo simulations in our numerical study of the two model systems. Both systems are kept at zero temperature. The algorithms used in the current work are described below.

For the systems governed by Eq. (1), all spins are updated simultaneously. A spin will flip if its local field $f_i=\Sigma J_{ij}S_j+h_i+H$ changes sign. The external field is decreased by dH and then is fixed to drive the avalanches until the system reaches a metastable state. The varying rate of magnetic field can be measured by dH. If dH is fixed throughout the magnetization reversal process, H is a linear varying field. If dH is adjusted to be the local field of the most unstable spin, H is a quasistatic driving field. Unless specified otherwise, the simulation results are obtained from the systems with infinitely slow driving rate.

To speed up the simulation, a fast algorithm similar to the sort-list algorithm [11] is used. This algorithm is not applicable to the 2D RBIM with Gaussian random bonds because of the possibility of antiferromagnetic exchange interaction. Physical quantities are averaged over 1000–50 000 configurations. The average depends on the system size. One time step, or one Monte Carlo step (MCS), in the simulation is defined as one attempt of all spin updates. When the driving

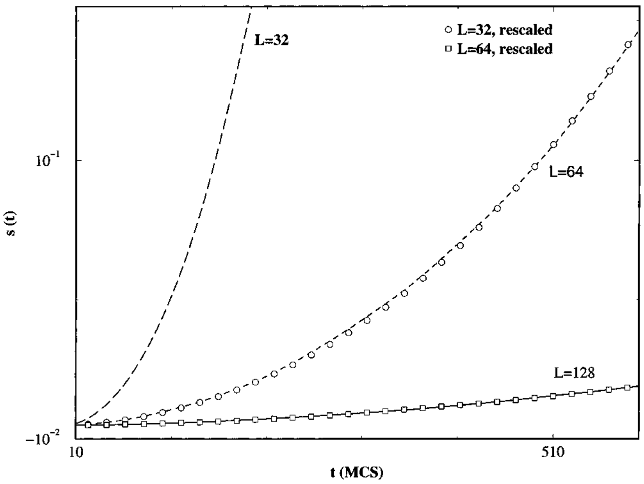

FIG. 3. Dynamic finite-size scaling for the largest avalanche at $D_c=2.16$ in 3D RFIM. The symbols are rescaled by Eq. (4) with $H=H_c$ and $D=D_c$.

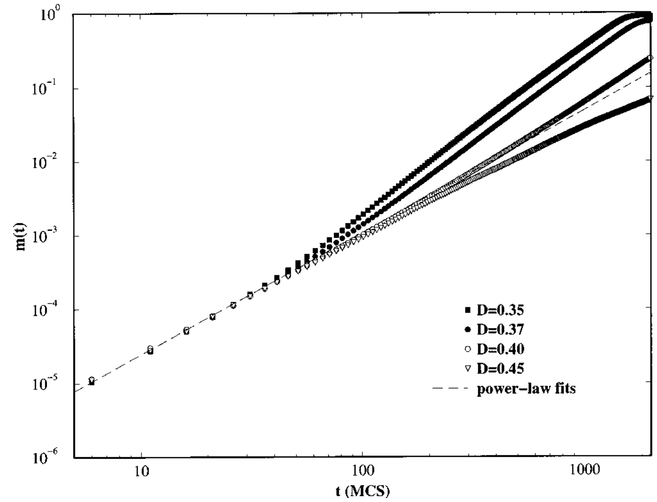

FIG. 4. The growth of the largest avalanche in a two-dimensional random-bond Ising model (2D RBIM); $L=1024$. The plots are in log-log scale. The dashed line is the best power-law fit.

field changes, the time is reset to zero.

B. Results

1. 3D random-field Ising model

Figure 1(a) shows the evolution of the largest avalanche at different disorder strength D in the 3D RFIM. The best power-law fit determines the critical value $D_c(L)$ in the finite system. Figure 1(b) shows the corresponding static critical scaling [Eq. (3)]. The critical value $D_c(\infty)$ of the infinite system can be determined from the relation

$$D_c(L)-D_c(\infty)\sim L^{-1/\nu}, \tag{7}$$

which is plotted in Fig. 2. The dynamic finite-size scaling for the avalanche size is shown in Fig. 3.

From the dynamic scaling of the largest avalanche [Eqs. (4)–(7)], the critical exponents β, ν, z, and the critical value D_c can be obtained. The values are listed in Table I. We compare these exponents and D_c obtained from the dynamic scaling with those determined from stationary scaling [Eqs. (2) and (3)] in 3D RFIM. As shown by the results in Table I, they are in fairly good agreement.

2. 2D random-bond Ising model

Figure 4 shows the evolution of the largest avalanche at different disorder strength D in the 2D random-bond Ising model. Exponents β, ν, z and the critical value D_c obtained from the dynamic scaling for the 2D RBIM are listed in Table II. As shown in Fig. 5, they are consistent with those determined by stationary scaling [10]. From the above results, we can see that in both the 3D RFIM and 2D RBIM, the scaling relation between the new dynamic exponent θ and other critical exponents, $\theta=[d-\beta/\nu]/z$, is indeed universal.

IV. DYNAMICAL HETEROGENEITY OF AVALANCHES

Equation (5) shows that the dynamics of the largest avalanche obeys a power-law scaling relation. This conclusion is true at least at the initial stage. At the point (D_c, H_c), which is the critical point of the disorder-induced phase transition, the relaxation time or the duration time of the avalanche becomes infinite. As shown earlier, this relation is valid for the 2D RFIM [14]. It is also confirmed presently in the 3D RFIM and 2D RBIM. Therefore, Eq. (5) describes the universal short-time dynamic scaling for the driven disordered systems.

The avalanches in driven disordered system have different sizes and duration times. For example, in magnetization reversal process, there are various Barkhausen jumps caused by different sizes of domains. Since Eq. (5) is based on the assumption that the avalanche occurs at the critical point and

TABLE I. The critical exponents for the disorder-driven phase transition in 3D RFIM. The exponents and D_c determined by different methods are listed for comparison.

	$D_c(\infty)$	$1/\nu$	β/ν	z	θ
Dynamic scaling	2.158 ± 0.005	1.1 ± 0.1	0.05 ± 0.02	1.68 ± 0.03	1.76 ± 0.02
Equilibrium scaling (Ref. [11])	2.16 ± 0.03	0.7 ± 0.1	0.02 ± 0.02	1.69	1.76[a]
Exact results (Ref. [11])		1	0		

[a]The value determined from $1/\sigma\nu z$.

TABLE II. The critical exponents in 2D RBIM determined by dynamical scaling and static scaling.

	$D_c(\infty)$	z	β	ν	θ
Dynamic scaling	0.40 ± 0.01	1.3 ± 0.1	0.05 ± 0.01	1.0 ± 0.1	1.50 ± 0.02
Equilibrium scaling (Ref. [10])	0.44 ± 0.03	1.2 ± 0.1	0.065 ± 0.1	1.4 ± 0.1	

the relaxation time of short-time regime is large, one may argue that the short-time dynamics and the exponent θ are not universal for those avalanches with small or finite sizes and times. It is therefore interesting to find out what the avalanches are of *all sizes*.

To resolve this issue, we measure the dynamic scaling exponent θ of each avalanche in 2D random-bond Ising model. From the power-law relation, we can calculate the exponent θ for an avalanche through the following relation:

$$\theta=\frac{\ln s_2-\ln s_1}{\ln \Delta t}, \tag{8}$$

where $\Delta t=t_2-t_1$, s_2 and s_1 are avalanche sizes at time t_2 and t_1, respectively, in an avalanche process. $t_1=2-5$ MCS and t_2 is taken to be $2\tau_0/3$. The value θ is measured for each avalanche and is averaged over 10^4-10^5 random bond configurations.

Figure 6 is the histogram plot of θ at different values of D. It can be seen that the major peak position of θ shifts from a small value θ_1 to a larger value θ_2 when D decreases. This behavior clearly indicates that there are two kinds of dynamical evolution mechanism. For D not far below D_c, two evolution processes coexist. We also find that θ_1 does not change when D varies, while θ_2 increases when D decreases. These results are consistent with the two-stage kinetics of fracture process observed early in the 2D RFIM model system [14].

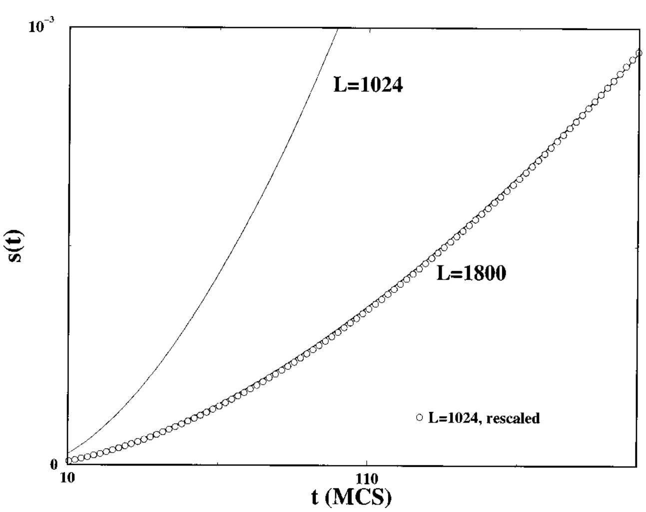

FIG. 5. Dynamic finite-size scaling for the largest avalanche at $D_c=0.41$ in 2D RBIM. The symbols are rescaled by Eq. (4) with $H=H_c$ and $D=D_c$.

To exclude the effect of distribution function $P(J_{ij})$ on the distribution of θ, we chose an uniform distribution function of $P(J_{ij})$. Again the same characteristic of the distribution $P(\theta)$ is found. The results in Fig. 6 strongly suggest that the driven disordered system is dynamically heterogeneous at $D<D_c$ for the avalanche evolution under homogeneous external field. However, in the short-time regime or small spatial regions, the kinetics of avalanche is homogeneous, i.e., there exists a universal dynamic scaling exponent θ_1 that is characteristic for the disordered systems.

V. DYNAMICS OF AVALANCHES IN MEAN-FIELD RFIM

Using finite-size scaling and Monte Carlo simulation in Sec. III, we found the universality of dynamic scaling for avalanche process in some driven disordered systems, i.e., 2D and 3D RFIM and 2D RBIM. It is important to look at the exact results for dynamic behavior of avalanches in some solvable disordered models. In RFIM, the kinetics of avalanche can be exactly solved using the mean-field approximation.

The single-spin-flip kinetics for a random-field Ising model described by Eq. (1) can be described by the master equation

$$\frac{d}{dt}P(\{S_j\},\ t)=\sum_j\ [w_j(-S_j)P(\{-S_j\},t)-w_j(S_j)P(\{S_j\},\ t)], \tag{9}$$

where $P(\{S_j\},t)$ is the probability of state $\{S_j\}\equiv\{S_1,\ \dots,\ S_j,\ \dots,\ S_n\}$ and $\{-S_j\}\equiv\{S_1,\ \dots,\ -S_j,\ \dots,\ S_n\}$

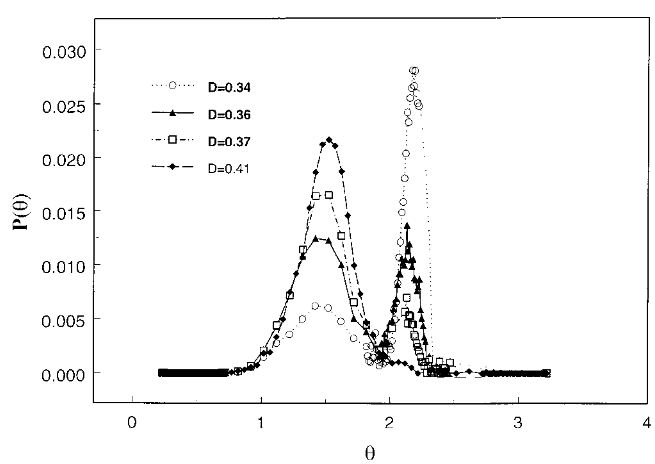

FIG. 6. The histogram plot of the distribution of exponent θ at different strengths of disorder in 2D RBIM; $L=1024$.

is the state generated by the flipping of spin S_j in state $\{S_j\}$. For detailed balance condition at finite temperature T, the transition rate is $w_j(S_j)=[1-S_j f(S_j)]/2$. $f(S_j)$ is a function of the nearest neighbor of the jth spin and can be written as

$$f(S_j)=\tanh\left[\left(J\sum_i S_i+h_i+H\right)\Big/ k_BT\right]. \tag{10}$$

For simplification, we assume that the random-field probability distribution function is a bimodal function:

$$P(h_j)=[\delta(h_j-h_0)+\delta(h_j+h_0)]/2, \tag{11}$$

where h_0 is the strength of random field. Based on mean-field approximation, the thermal average magnetization $\langle S\rangle$ is independent of site j, and Eq. (9) leads to

$$\frac{d}{dt}\langle S_j\rangle=-\langle S_j\rangle+\langle f(S_j)\rangle. \tag{12}$$

Since the observed magnetization can be written as $m=[\langle S_j\rangle]$, where $[\cdots]$ denotes the average over the random-field configurations, Eq. (12) therefore becomes

$$\frac{d}{dt}m=-m+\frac{1}{2}\left[\tanh\left(\frac{Jm+h_0+H}{k_BT}\right)+\tanh\left(\frac{Jm-h_0+H}{k_BT}\right)\right]. \tag{13}$$

It is well known that for $H=0$ and without the random fields, the mean-field Ising model has a second-order phase transition. The presence of random field $\{h_i\}$ may introduce a tricritical point to the phase diagram. To simplify the kinetics governed by Eq. (13), we just consider the system that has a second-order phase transition when H is not applied. The applied field can drive the system to a metastable state before the breakdown of the system, i.e., sign change of m. We assume that (H_s, m_s) is the end point of the metastable state in the magnetization curve for a given temperature T below the critical temperature, or in other words, the spinodal point. The relation between H_s and m_s can be obtained from the equilibrium relation in Eq. (13),

$$m_s=\frac{1}{2}\left[\tanh\left(\frac{Jm_s+h_0+H_s}{k_BT}\right)+\tanh\left(\frac{Jm_s-h_0+H_s}{k_BT}\right)\right]. \tag{14}$$

Let $\Delta m(t)=m(t)-m_s$ be the evolution of avalanche at $H=H_s$. Since at the beginning, $\Delta m(t)$ is small, Eq. (13) can be written as

$$\frac{d}{dt}\Delta m=-\Delta m+\left[\sec h^2\left(\frac{Jm_s+H_s+h_0}{k_BT}\right)+\sec h^2\left(\frac{Jm_s+H_s-h_0}{k_BT}\right)\right]\Delta m. \tag{15}$$

Therefore the evolution of $\Delta m(t)$, which is the solution to Eq. (15), can be written as

$$\frac{\Delta m(t)}{\Delta M}=1-\exp(-t/\tau), \tag{16}$$

where ΔM is the Barkhausen jump in the magnetization curve and τ is the characteristic time:

$$\tau=\left[1-\sec h^2\left(\frac{Jm_s+H_s+h_0}{k_BT}\right)-\sec h^2\left(\frac{Jm_s+H_s-h_0}{k_BT}\right)\right]^{-1}. \tag{17}$$

Varying the strength of disorder h_0 can make $\tau\to\infty$, as shown by Eq. (17), which is consistent with the evolution of the largest avalanche in the RFIM discussed in the simulation studies in Sec. III. Therefore when $\tau\to\infty$, from Eq. (16) we have $\Delta m(t)\propto t$ in short times, or the breakdown process can be characterized by a power law in time with the exponent $\theta=1$.

VI. DISCUSSION AND CONCLUSIONS

In our previous investigation in a 2D random-field Ising model, we observed the scaling relation for the dynamic avalanche process. The short-time dynamic shows universal scaling for the largest or spanning avalanches [14]. However, this study does not give the general answer to the questions critical to our understanding of the nature of the dynamic processes in driven disordered systems. One of these questions is whether the scaling relation, as observed in the 2D RFIM, is universal in higher-dimensional systems, and also for systems without hyperscaling. To resolve these issues, we carried out Monte Carlo simulations on two separate model systems. One is a three-dimensional random-field Ising model and another is a two-dimensional random-bond Ising model.

The second question is whether or not the kinetics of all avalanche processes are homogeneous. Specifically, we are interested in finding out how avalanches of *all sizes* behave, not just the largest or spanning one as we studied earlier [14].

Our simulations on the avalanches in both systems, the 3D RFIM and the 2D RBIM, show that the evolution driven by the sweeping field indeed exhibits dynamic scaling. Moreover, the dynamics of avalanches is heterogeneous, i.e., there are two different kinetics involved and the dynamics is affected by the disorder strengths. Two evolution mechanisms contribute to the dynamic heterogeneity. At the initial stage of evolution, the avalanche is a result of diffusive process, and the dynamic scaling exponent is small. Small avalanches appear randomly throughout the system, and the exponent is universal for avalanches at short times. For the avalanche with larger dynamic scaling exponent, its growth process is a collective phenomenon. There is a strong correlation among local events during such a process. The same conclusions are drawn for the 2D RFIM from our early work [14].

The dynamic scaling for avalanches in *short times* is indeed universal for the driven disordered systems. As shown in the 3D RFIM and the 2D RBIM, the scaling relation is obeyed in three dimensions and in the system where the hyperscaling does not hold. In addition, the scaling exponent

θ is independent of the avalanche size.

One of the benefits from the universality is that the exponent θ can serve as a fingerprint to characterize the disordered system. The dynamical short-time scaling can be used as an efficient alternative to analyze the critical properties of the disordered system. For example, in disordered magnetic materials, dynamic measurements of Barkhausen avalanche can be carried out in short-time regime, and no particular size of avalanche is required.

The dynamics of avalanches is also analyzed in a mean-field RFIM at finite temperature. At short times, the size of the Barkhausen jump is proportional to time.

In conclusion, we demonstrate by numerical simulations of both random-field and random-bond Ising models that avalanches in driven disordered system have universal short-time dynamic scaling. The dynamic scaling is valid for both high-dimensional systems and systems without hyperscaling. The new dynamic exponent θ is shown to be related to other critical exponents: $\theta=[d-\beta/\nu]/z$. Dynamic heterogeneous nature of avalanches, which is affected by the strength of disorder, is also observed.

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